Power Randomization for Iterative Detection Over Random-Access Fading Channels

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***Abstract*—In this paper, we focus on throughput performance of multiuser communications systems over fading channels. More specifically, we consider the uplink where multiuser detection under asynchronous transmissions is exploited for random-access management and iterative receivers are considered for practical issues. Normalized throughput is evaluated through a semi-analytic procedure in order to avoid time-consuming simulations. Power randomization is explored as a means for improving performance through system asymmetries. It is found beneficial, particularly in case of overloaded systems operating at low-to-medium signal-to-noise ratio, where it allows reducing significantly the number of iterations at the receiver.**

***Index Terms*—Code-division multiple access (CDMA), fading channels, iterative decoding, multiuser detection, power randomization, random-access communications, throughput evaluation.**

I. Introduction

Wireless networks need to be capable of serving a large number of mobile users which share limited resources. Among the techniques for resource sharing, *random access* does not require any coordination by the users. This is especially beneficial in satellite communications [1] where signaling delays make user coordination and the use of feedback in access difficult. Commonly, random access communications have been developed and analyzed through collision resolution in the context of networking and through multiaccess channels in the context of information theory [2]. However, the gap between networking (traditionally focusing on the randomness of packet arrivals) and information theory (traditionally focusing on the interference and noise), well described in [3], is still not bridged.

To this aim, different theoretical approaches have been proposed in the recent literature. Packet-based random multiple access with bursty traffic has been analyzed in [4] through a generalization of the classical information theoretic analysis providing an achievable region in terms of communication rates with reliable collision detection. Users select their communication rate independently without sharing such information with the other users or with the receiver. Deterministic interference model in order to provide transmission schemes for random access with a provable gap to capacity have been analyzed in [5]. Multiple superimposed streams are transmitted (broadcast), and a subset is decoded at the receiver. Also, cross-layer approaches exploiting signal processing techniques at PHY-layer for advanced MAC-layer design have been explored in [6].

The inherent characteristic of random access is, naturally, the random load of the system resulted by the random number of users that at any instant attempt to access. Broadly speaking, the information theoretic principle is that the receiver is able to decode the instantaneous load if it lies within the capacity region of the multiple access channel, otherwise not. If the total number of users attempting to access is small, as happens in a narrowband system, the fluctuation can be significant. One simple way to reduce the fluctuation is to utilize broadband signaling, e.g., Code Division Multiple Access (CDMA), and enabling more users to access the same broadband resource. In an asymptotic system each of the infinitely many users transmits independently an infinitely small data packet, while the system load is deterministic. As an added benefit, CDMA is able to deal with user asynchronism in a simple way, and allows the system to be designed with much looser timing requirements than corresponding narrowband, e.g., Time Division Multiple Access (TDMA), counterparts. Asynchronism is a key issue in system design as it may be exploited as the main source for user separation at the receiver location. Detailed analysis of the benefit due to asynchronism is found in [7], [8] for CDMA systems with both optimal and suboptimal linear multiuser receivers.

Analysis of the spectral efficiency in CDMA systems with large number of users, and its convergence properties, is developed in [9] with a focus on system load, i.e., the ratio between the number of users and the number of dimensions. The effects of the choice of the signatures and of the power control on the loss in spectral efficiency is also considered.

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Power control, and specifically random power control, has traditionally been utilized in random access to provide the capture effect [10]. The impact of finite order modulation and realistic codes on system capacity is studied in [11], where systems with Quadrature Phase Shift Keying (QPSK) and binary codes are shown to approach the capacity of Gaussian signaling as the size of the user population grows. Therein, it is also made clear that unequal power allocation will help a multiple access system to operate within the capacity region. Focusing on low-complexity iterative receivers, e.g., based on (linear) minimum mean square error (MMSE) filters, system asymmetries represent a solution for practical convergence issues in order to achieve near-optimal performance [12]. The convergence of certain MMSE based interference cancellation receivers can be evaluated semi-analytically with the help of extrinsic information exchange principles [12], [13]. It is worth mentioning that asynchronous CDMA systems with random access and on-off keying have been recently proposed for control channels and scheduling request modeling in cellular systems [14]. Some recent works have focused on random access through spread spectrum in satellite networks where power randomization is found beneficial [15], [16].

In this paper we analyze the performance, in terms of normalized throughput, of a random access wireless communications system using CDMA and a simple ALOHA type random access protocol. More specifically, asynchronous communications are assumed in order to simplify the random access and iterative receivers are assumed for practical implementation issues (i.e., low-complexity approach). Power randomization is explored as a means to enforce system asymmetries which aid iterative receiver processing and avoiding stringent requirements such as channel state information (CSI) at the transmitter. The spectral efficiency is considered as a benchmark for comparison. Performance are evaluated with reference to two fading models (Rayleigh and log-normal) in order to gain insight on performance over terrestrial and satellite wireless links.

The main paper contributions are:

- presenting a semi-analytical approach for throughput performance evaluation of asynchronous multiuser communications in the uplink with random access;
- evaluate the impact on the performance of the instantaneous system load and of power randomization.

Notation: Lower-case bold letters denote vectors, with a_n denoting the nth element of $\boldsymbol{a}$; upper-case bold letters denote matrices, with $A_{n,m}$ denoting the (n, m)th element of $\boldsymbol{A}$; $\star$ denotes convolution; $\boldsymbol{I}_N$ denotes the $N \times N$ identity matrix; $\mathrm{diag}(\boldsymbol{a})$ denotes a diagonal matrix with $\boldsymbol{a}$ on the main diagonal; $\mathbb{E}\{\cdot\}$, $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, and $(\cdot)^{\mathrm{H}}$ denote expectation, conjugate, transpose and conjugate transpose operators; $\det(\boldsymbol{A})$ and $\mathrm{tr}(\boldsymbol{A})$ represent the determinant and the trace of $\boldsymbol{A}$, respectively; $\Pr(\mathcal{A})$ denotes the probability of the event $\mathcal{A}$; $p_a(\cdot)$ denotes the probability density function (PDF) of the random variable a; j is the imaginary unit; $\mathcal{N}(\mu, \sigma^2)$ denotes a real-valued normal distribution with mean μ and covariance σ^2; $\mathcal{N}_{\mathbb{C}}(\mu, \sigma^2)$ denotes a complex-valued proper normal distribution with mean μ and covariance σ^2; $\mathcal{Q}(\cdot)$ denotes the upper tail of a zero-mean unitary-variance real-valued normal distribution; $\sim$ means "distributed as."

II. System Model

We consider a system in the uplink with variable number of users, i.e., in an arbitrary time interval each user may be active or not. Users transmit their own information, coded by using a rate-R forward-error-correction code and QPSK modulation, through packets of L symbols. One single symbol has duration T_s, thus a packet is allocated into an interval of duration LT_s.

If active, the kth user transmits the following baseband waveform for the mth symbol

$$s_k(t; m) = b_k[m] \sum_{n=0}^{N-1} c_k[n] \psi_{\mathrm{tx}}(t - nT_c - mT_s - \tau_k), \qquad (1)$$

where $b_k[m]$ represents the mth QPSK symbol of the kth user, otherwise remains silent if inactive. The notation in (1) is common in CDMA literature, where $\boldsymbol{c}_k = (c_k[0], \ldots, c_k[N-1])^{\mathrm{T}}$ denotes the signature sequence for the kth user, T_c is the chip duration, and N is the spreading factor. The number of signature sequences available for the whole set of users is denoted M: we will see that a few sequences are sufficient. The set of delays $\{\tau_0, \ldots, \tau_{K-1}\}$ takes into account for users asynchronism, where time t is measured with respect to the clock at the common receiver. Also, $\psi_{\mathrm{tx}}(t)$ takes into account for the shape of the transmit filter.

Without loss of generality, we will assume $0 = \tau_0 < \tau_1 < \ldots < \tau_{K-1} < T_c$. It has been shown that any shift (of the signal) of an integer multiple of T_c does not change the average system performance [7], [8]. The system may be viewed as chip-asynchronous and symbol-synchronous.

A. Signal Model

The signal received at the common receiver from transmission of K active users is

$$y(t; m) = \sum_{k=0}^{K-1} h_k[m] \sqrt{P_k} s_k(t; m) \star \psi_{\mathrm{rx}}(t) + w(t), \qquad (2)$$

where $h_k[m]$ denotes the fading coefficient for the kth user over the whole mth symbol duration (taking into account the small-scale fading), P_k represents the received power for the kth user (taking into account large-scale fading and mismatch in power control at receiver location), $\psi_{\mathrm{rx}}(t)$ denotes the receive filter and $w(t)$ is an additive Gaussian noise with flat power spectral density (N_o) within the receiver bandwidth. For sake of simplicity, shapes of transmit and receive filters are combined in the single elementary baseband waveform

$$\psi(t) = \psi_{\mathrm{tx}}(t) \star \psi_{\mathrm{rx}}(t). \qquad (3)$$

Inter-symbol interference may be neglected if the spreading factor is large enough ($N \gg 1$). A sampling frequency $f_s = r/T_c$, i.e., using an oversampling factor (r) with respect to the chip rate, provides the following discrete-time model of size $rN \times K$ to be processed in order to recover the information vector $\boldsymbol{b}[m]$:

$$\boldsymbol{y}[m] = \boldsymbol{A}[m]\boldsymbol{b}[m] + \boldsymbol{w}[m], \qquad (4)$$

where

$$\boldsymbol{y}[m] = \left(y_0[m], \ldots, y_{r(N-1)}[m]\right)^{\mathrm{T}}, \tag{5}$$
$$\boldsymbol{b}[m] = \left(b_0[m], \ldots, b_{K-1}[m]\right)^{\mathrm{T}}, \tag{6}$$
$$\boldsymbol{w}[m] = \left(w_0[m], \ldots, w_{r(N-1)}[m]\right)^{\mathrm{T}}, \tag{7}$$

being

$$y_i[m] = y(t)|_{t=mT_s+i/f_s}, \tag{8}$$
$$w_i[m] = w(t)|_{t=mT_s+i/f_s} \sim \mathcal{N}_{\mathbb{C}}\left(0, \sigma_w^2\right), \tag{9}$$

with $\sigma_w^2 = N_o r/T_c$ and where

$$\boldsymbol{A}[m] = \boldsymbol{\Psi}[m] \cdot \boldsymbol{H}[m] \cdot \boldsymbol{P}^{1/2}, \tag{10}$$

with

$$\boldsymbol{\Psi}[m] = \begin{pmatrix} \boldsymbol{\Psi}_{0,0}[m] & \cdots & \boldsymbol{\Psi}_{0,K-1}[m] \\ \vdots & \ddots & \vdots \\ \boldsymbol{\Psi}_{r(N-1),0}[m] & \cdots & \boldsymbol{\Psi}_{r(N-1),K-1}[m] \end{pmatrix}, \tag{11}$$

taking into account for the contributions of the chip-modulated delayed elementary waveforms, being

$$\boldsymbol{\Psi}_{i,k}[m] = \sum_{n=0}^{N-1} c_k[n]\psi\left(iT_c/r - nT_c - \tau_k\right), \tag{12}$$

with

$$\boldsymbol{H}[m] = \operatorname{diag}\left(h_0[m], \ldots, h_{K-1}[m]\right), \tag{13}$$

taking into account for the contributions of the small-scale fading, and with

$$\boldsymbol{P} = \operatorname{diag}(P_0, \ldots, P_{K-1}), \tag{14}$$

taking into account for the contributions of the large-scale fading and power control mismatches. In the rest of the paper, if not necessary, we will omit explicit dependence on the time-slot index m.

The (instantaneous) system load and the signal-to-noise ratio (SNR) are defined as

$$\alpha = \frac{K}{N}, \qquad \gamma = \frac{\operatorname{tr}\left(\boldsymbol{A}\boldsymbol{A}^{\mathrm{H}}\right)}{K\sigma_w^2}, \tag{15}$$

respectively.

B. Channel Model

We assume two different channel models for fading:

- Rayleigh fading, this model is suitable for terrestrial channels[1];
- Log-normal fading, this model is suitable for satellite channels.

[1]A more general model for terrestrial channels is the Nakagami fading [17], which includes Rayleigh fading as a special case. Rayleigh fading can be considered realistic for *non-line-of-sight* scenarios.

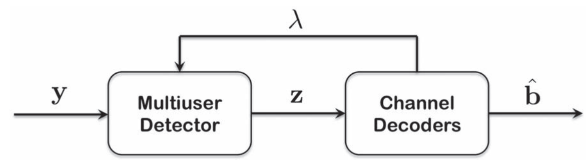


Fig. 1. Block diagram of the iterative receiver.

In the first case, the coefficients $g_k = |h_k|^2$ are assumed i.i.d. undergoing an exponential distribution with parameter μ_g, i.e., their PDF is

$$p_{g_k}(\xi) = \frac{1}{\mu_g} \exp\left(-\frac{\xi}{\mu_g}\right) (\xi \geq 0). \tag{16}$$

In the second case, the coefficients $g_k = |h_k|^2$ are assumed i.i.d. undergoing a log-normal distribution with parameters $\tilde{\mu}_g$ and $\tilde{\sigma}_g$, in dB, i.e., their PDF is

$$p_{g_k}(\xi) = \frac{10/\log(10)}{\sqrt{2\pi}\tilde{\sigma}_g\xi} \exp\left(-\frac{\left(10\log_{10}(\xi) - \tilde{\mu}_g\right)^2}{2\tilde{\sigma}_g^2}\right). \tag{17}$$

C. Power Randomization

It is known that iterative receivers benefit largely from system asymmetries which could be optimally designed by modifications of power control procedures [11], [12]. However, when feedback at the transmitter is not possible (as assumed in this work), one possibility to force asymmetry in the system is to apply power randomization: each user select his transmit power by drawing samples from a given PDF. Power randomization also makes the operation point to lie further within the capacity region [11], thus helping the iterative process.

We consider the case in which the power coefficients P_k are assumed uniformly distributed[2] in logarithmic scale in the range $(-\Delta_p$ dB, $+\Delta_p$ dB). It is worth noticing that the system not employing power randomization (i.e., $P_k = 1$) is labeled with $\Delta_p = 0$ dB.

III. Receiver Structure

As shown in Fig. 1, the receiver is made of two main building blocks:

- a multiuser detector, aiming at unveiling the information streams coming from each of the different users;
- a bank of soft-input soft-output (SISO) channel decoders, aiming at decoding the source information from the single stream.

It is worth mentioning that here we assume perfect CSI at the receiver, while practical receivers do implement channel estimation which may be placed within the iterative loop and benefit from the turbo effect [18], [19].

[2]A uniform distribution can easily incorporate peak-power constraints.

A. Multiuser Detector

Let us recall that MMSE filtering means that the received signal in (2) is processed as

$$\boldsymbol{z} = \left(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A} + \sigma_w^2 \boldsymbol{I}_K\right)^{-1} \boldsymbol{A}^{\mathrm{H}}\boldsymbol{y}. \tag{18}$$

The symbol z_k, related to the kth user, has SINR [20]–[22]

$$\zeta_k = \frac{1}{Q_{k,k}} - 1, \tag{19}$$

with $Q_{k,k}$ denoting the kth diagonal element of

$$\boldsymbol{Q} = \left(\boldsymbol{I}_K + \frac{1}{\sigma_w^2}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{A}\right)^{-1}. \tag{20}$$

We compare the performance of receivers based on MMSE filtering, which employ soft iterative successive interference cancellation (SIC) with users ordered according to their instantaneous strength. When decoding the information related to the kth user, SIC replaces the received signal model in Eq. (4) with

$$\boldsymbol{y}_{(k)} = \boldsymbol{y} - \boldsymbol{A}\hat{\boldsymbol{b}}_{(k)}, \tag{21}$$

where $\hat{\boldsymbol{b}}_{(k)} = (\hat{b}_1, \ldots, \hat{b}_{k-1}, 0, \ldots, 0)^{\mathrm{T}}$ and $\hat{b}_k$ denotes the estimate for b_k provided by the channel decoders.

More specifically, the standard MMSE equations are modified on the basis of the SIC procedure, i.e., the symbol z_k and its corresponding SINR ζ_k are obtained replacing $\boldsymbol{A}$ with $\boldsymbol{A}\boldsymbol{E}_{(k)}$. Also, $\boldsymbol{E}_{(k)}$ is a diagonal matrix iteratively updated with respect to $\boldsymbol{E}_{(k-1)}$ by replacing the $(k-1)$th diagonal element with $\sqrt{1 - \mathbb{E}\{|\hat{b}_{k-1}|^2\}}$ and starting from $\boldsymbol{E}_{(1)} = \boldsymbol{I}_K$, or briefly

$$\boldsymbol{E}_{(k)} = \mathrm{diag}\left(e_{(k),0}, \ldots, e_{(k),K-1}\right), \tag{22}$$

$$e_{(k),m} = \begin{cases} \sqrt{1 - \mathbb{E}\left\{|\hat{b}_m|^2\right\}} & 0 \le m < k \\ 1 & k \le m \le K-1. \end{cases} \tag{23}$$

Performance improvements are obtained if the SIC procedure is repeated: we denote I the total number of iterations. More specifically, the symbol and its corresponding SINR of the kth user at the ith iteration, denoted $z_{k;(i)}$ and $\zeta_{k;(i)}$, respectively, are obtained replacing $\boldsymbol{A}$ with $\boldsymbol{A}\boldsymbol{E}_{(k;i)}$. Denoting $\hat{b}_{k,(i)}$ the estimate for b_k at the ith iteration, we have

$$\boldsymbol{E}_{(k;i)} = \mathrm{diag}\left(e_{(k;i),0}, \ldots, e_{(k;i),K-1}\right), \tag{24}$$

$$e_{(k;i),m} = \begin{cases} \sqrt{1 - \mathbb{E}\left\{\left|\hat{b}_{m;(i)}\right|^2\right\}} & 0 \le m < k \\ 1 & m = k \\ \sqrt{1 - \mathbb{E}\left\{\left|\hat{b}_{m;(i-1)}\right|^2\right\}} & k < m \le K-1. \end{cases} \tag{25}$$

In contrast to Eq. (23), which is used only for the first iteration, results from the previous iteration are exploited in Eq. (25) for assessing the contribution of the users still to be canceled.

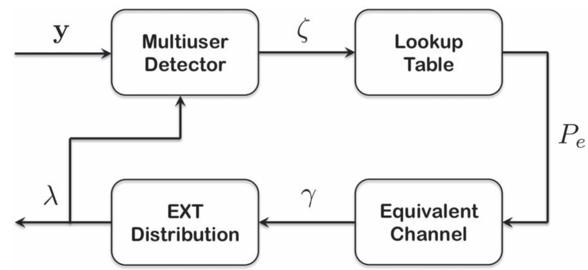


Fig. 2. Block diagram of the semi-analytical approach for performance evaluation.

B. Channel Decoder

At the ith iteration, the decoder is based on the extrinsic log-likelihood

$$\lambda_{k;(i)} = \log\left(\frac{\Pr\left(b_k = 1|z_{k;(i)}\right)}{\Pr\left(b_k = 0|z_{k;(i)}\right)}\right), \tag{26}$$

which is fed back to the multiuser detector for soft SIC through

$$\hat{b}_{k;(i)} = \tanh\left(\lambda_{k;(i)}/2\right). \tag{27}$$

The exact computation of the extrinsic information depends on the specific channel code.

IV. PERFORMANCE-EVALUATION METHODOLOGY

A. Normalized Throughput

We adopt a semi-analytical approach in order to provide good predictions of the actual system performance without relying on heavy numerical simulations. Fig. 2 shows the block diagram of the considered approach.

Firstly, the SINR (ζ) is computed for each user through (19), (23), and (25), as explained in the previous section. Secondly, the mapping between the SINR (ζ) entering the SISO decoder and the corresponding BER (denoted P_e) is assumed to be known through a deterministic lookup table denoted $P_e(\zeta)$. Thirdly, a simplified model for the SISO decoder is considered in order to characterize the reliability of the estimates of the encoded bits.

More specifically, the extrinsic log-likelihood is modeled as

$$\lambda_{k,(i)} \sim \mathcal{N}\left(\mu_{k,(i)} b_k, 2\mu_{k,(i)}\right), \tag{28}$$

where the parameter $\mu_{k,(i)}$ defines an equivalent Gaussian channel which the symbol b_k is observed from. It is apparent that the equivalent Gaussian channel has the following equivalent SNR:

$$\gamma_{k;(i)} = \frac{\mu_{k;(i)}}{4}. \tag{29}$$

We assume that the BER (P_e) at the SISO decoder is related to the extrinsic information and its equivalent SNR ($\gamma_{k;(i)}$) through the following relation:

$$P_e = \mathcal{Q}\left(\sqrt{2\gamma_{k,(i)}}\right), \tag{30}$$

then combining Eqs. (29) and (30) we get

$$\mu_{k,(i)} = 2\left(\mathcal{Q}^{-1}\left(P_e\left(\zeta_{k,(i-1)}\right)\right)\right)^2. \tag{31}$$

The knowledge of the parameter $\mu_{k,(i)}$ is exploited by drawing samples from the distribution in Eq. (28) and in order to compute $\mathbb{E}\{|\hat{b}_{k;(i)}|^2\}$ through Eq. (27).

Finally, the mapping between the FER (denoted P_w) provided by the SISO decoder and the corresponding input SINR (ζ) is assumed to be known through a deterministic lookup table, as well. The normalized throughput at the ith iteration is given as

$$\theta_{(i)} = \frac{2R}{N}\sum_{k=0}^{K-1}\left(1 - P_w\left(\frac{\zeta_{k;(i)}}{2}\right)\right). \tag{32}$$

B. Spectral Efficiency

As a benchmark for throughput evaluation, we consider the average spectral efficiency (C), which requires the knowledge of the statistical distribution of the channel matrix, i.e.,

$$C = \mathbb{E}\{C(\boldsymbol{A})\}. \tag{33}$$

where the conditional spectral efficiency [21]–[23] is

$$C(\boldsymbol{A}) = \frac{1}{N}\log_2\left(\det\left(\boldsymbol{I}_{rN} + \frac{1}{\sigma_w^2}\boldsymbol{A}\boldsymbol{A}^{\mathrm{H}}\right)\right) \tag{34}$$

$$= \frac{1}{N}\sum_{\ell=1}^{\mathrm{rank}(\boldsymbol{A})}\log_2\left(1 + \gamma\rho_\ell^2\right), \tag{35}$$

and where $(\rho_1, \ldots, \rho_{\mathrm{rank}(\boldsymbol{A})})$ denote the *normalized* singular values of $\boldsymbol{A}$, i.e., the singular values of the matrix $(\sqrt{K/\mathrm{tr}(\boldsymbol{A}\boldsymbol{A}^{\mathrm{H}})})\boldsymbol{A}$.

V. Simulation Results

Here we show the performance of various system configurations obtained through numerical simulations performed with MATLAB. The evaluation metric is the normalized throughput (θ) as a function of the system load (α) and SNR (γ).

Both the number of available signature sequences and the oversampling factor can increase the achievable spectral efficiency (the larger, the better), however the improvement is not significant in terms of achievable spectral efficiency [7], [8], We considered chip sequences with $c_k[n] \in \{\pm 1/\sqrt{N}\}$ randomly generated with uniform distribution for the generic chip symbol, i.e., $\Pr(c_k[n] = +1/\sqrt{N}) = \Pr(c_k[n] = -1/\sqrt{N}) = 1/2$. Also, results are presented for spreading factor $N = 16$ and oversampling factor $r = 1$. It is worth noticing that, by sharing the same signature sequence, the users interfere in time, frequency, and code domains. Working with a small number of available signature sequences (M) means employing a receiver with the same small number of correlators. For sake of simplicity, a sinc-pulse is considered as elementary waveform, i.e.

$$\psi(t) = \frac{1}{\sqrt{T_c}}\mathrm{sinc}\left(\frac{t}{T_c}\right). \tag{36}$$

Codewords are assumed to be generated by a 3GPP2 turbo code with rate $R = 1/3$. Performance reported in the following can be considered valid for different values of the blocklength.

Fig. 3 shows the normalized throughput (θ) with respect to the SNR (γ) for $M = 1$, $M = 2$, and $M = 3$ available signature sequences for three system loads ($\alpha = 1/2$, $\alpha = 1$, and $\alpha = 3$) in both scenarios with Rayleigh fading and log-normal fading.

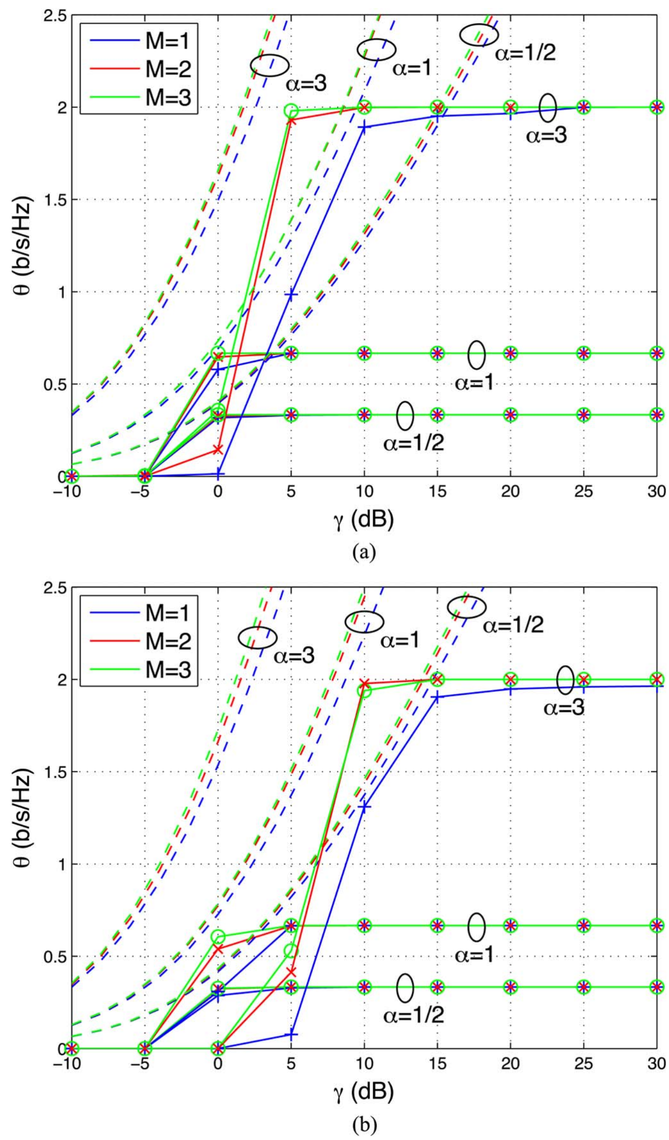


Fig. 3. Average spectral efficiency (C, dashed lines) and normalized throughput (θ, solid lines) as function of the SNR (γ) for different system loads (α) and number of signature sequences (M). No power randomization ($\Delta_p = 0$ dB) is employed at the transmitters and $I = 10$ iterations are employed at the receiver. (a) Rayleigh fading. (b) Log-normal fading.

The corresponding spectral efficiency (C) for the same system loads (α) is shown as a benchmark for comparison. It is apparent how the availability of $M = 2$ signature sequences provides significant performance improvements for overloaded systems, while adding more sequences does not provide any significant benefit. In the following we will focus on systems with $M = 2$ available signature sequences.

Fig. 4 shows the normalized throughput (θ) with respect to the SNR (γ) at $I = 1$ and $I = 10$ iterations for three system loads ($\alpha = 1/2$, $\alpha = 1$, and $\alpha = 3$) in both scenarios with Rayleigh fading and log-normal fading. Again, the corresponding spectral efficiency (C) for the same system loads (α) is shown as a benchmark for comparison. It is apparent how the turbo effect of the iterative receiver may push the performance very close to the spectral efficiency for a limited SNR range. Also, it is worth noticing how the turbo effect is necessary in

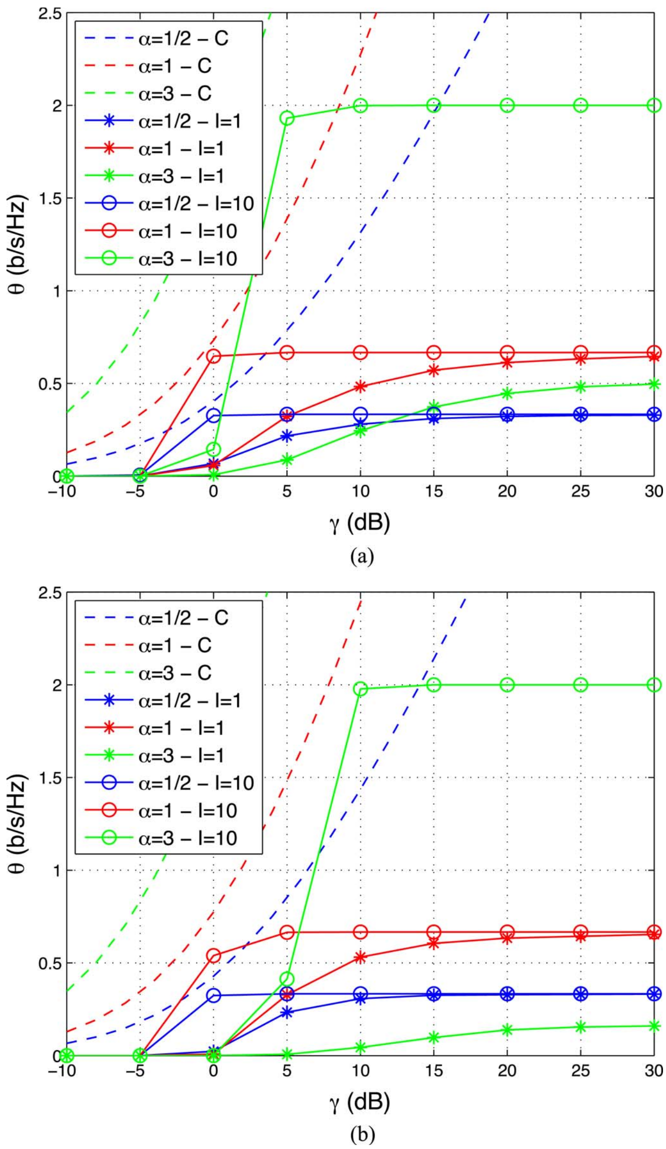


Fig. 4. Average spectral efficiency (C, dashed lines) and normalized throughput (θ, solid lines) as function of the SNR (γ) for different system loads (α) and numbers of iterations (I). No power randomization is employed ($\Delta_p = 0$ dB). (a) Rayleigh fading. (b) Log-normal fading.

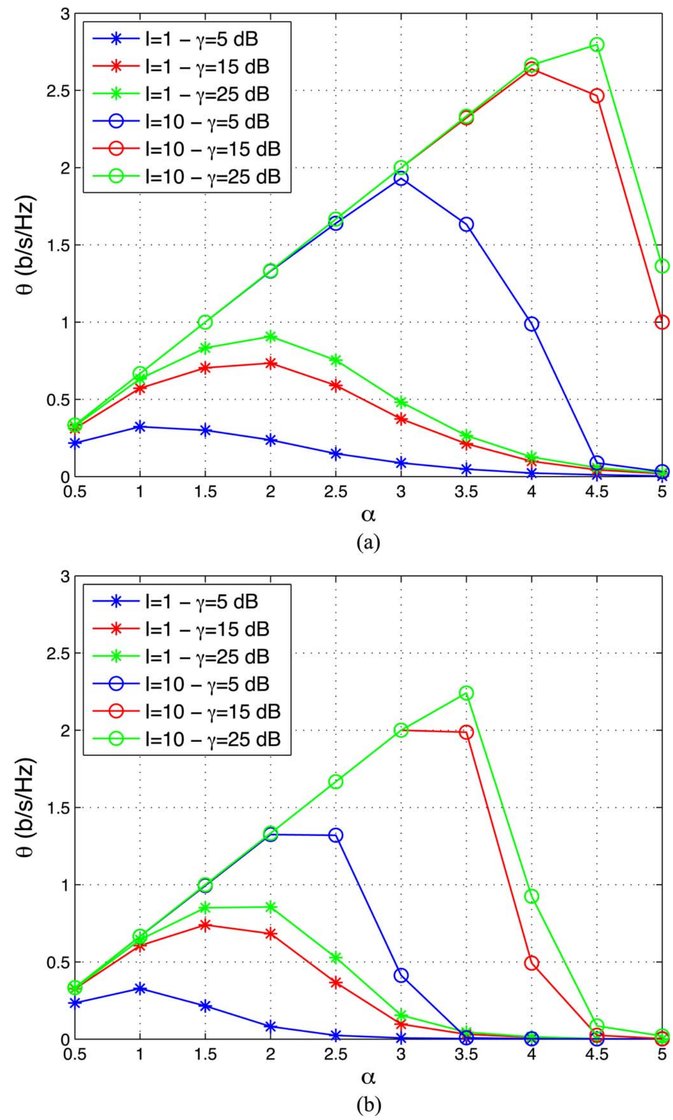


Fig. 5. Normalized throughput (θ) as function of the system load (α) for different SNR (γ) and numbers of iterations (I). No power randomization is employed ($\Delta_p = 0$ dB). (a) Rayleigh fading. (b) Log-normal fading.

TABLE I
MINIMUM GAP BETWEEN C AND θ ($I = 10$ AND $\Delta_p = 0$ dB)

Fading \ α	$1/2$	1	3
Rayleigh	1.4 dB	0.9 dB	3.5 dB
Log-normal	1.8 dB	2.3 dB	8.5 dB

order to exploit the capacity increase offered by large loads which differently destroy performance of the system with one single iteration.

Furthermore, it is apparent how the benchmark is practically identical for both fading scenarios, while the capacity loss (i.e., the gap between C and θ) is more pronounced for the scenario with log-normal fading than for the scenario with Rayleigh fading. This is summarized in Table I.

Fig. 5 shows the normalized throughput (θ) with respect to the system load (α) at $\gamma = 5$ dB (low SNR), $\gamma = 15$ dB (medium SNR) and $\gamma = 25$ dB (high SNR) in both scenarios with Rayleigh fading and log-normal fading. It is apparent how the iterative receiver with $I = 10$ iterations operating at $\gamma = 15$ dB is able to provide linear throughput scaling as a function of the offered system load up to 2.64 b/s/Hz (resp. 2.00 b/s/Hz) under Rayleigh (resp. log-normal) fading.

Previous results referred to the case in which no power randomization had been employed ($\Delta_p = 0$ dB). We now consider analogous performance results when employing power randomization ($\Delta_p = 6$ dB).

Fig. 6 shows how power randomization at the transmitters improves further the system performance. More specifically, it shows the same curves for normalized throughput (θ) with respect to the SNR (γ) only at $I = 10$ iterations (again for three system loads $\alpha = 1/2, \alpha = 1$, and $\alpha = 3$ in both scenarios with Rayleigh fading and log-normal fading) with and without power randomization (i.e., $\Delta_p = 0$ dB and $\Delta_p = 6$ dB). It is apparent to notice how the impact of power randomization is negligible for underloaded and fully-loaded systems. Also, not shown here for brevity, the impact of power randomization is also negligible when the number of iterations is small.

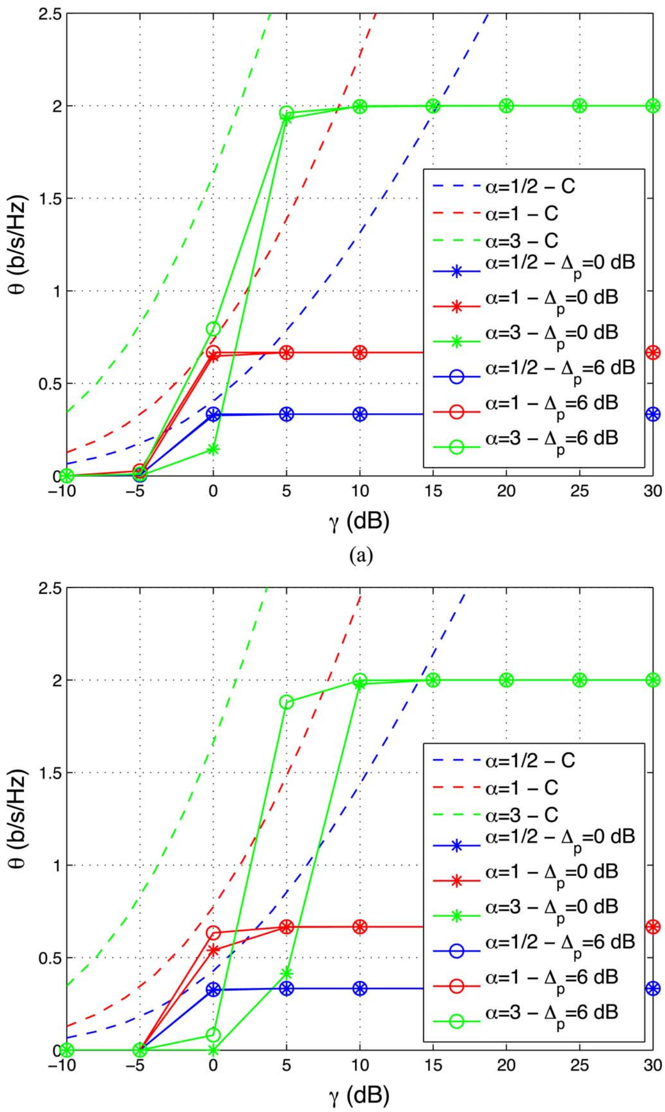


Fig. 6. Average spectral efficiency (C, dashed lines) and normalized throughput (θ, solid lines) as function of the SNR (γ) for different system loads (α): effect of power randomization with $\Delta_p = 0$ dB and $\Delta_p = 6$ dB on systems with $I = 10$ iterations at the receiver. (a) Rayleigh fading. (b) Log-normal fading.

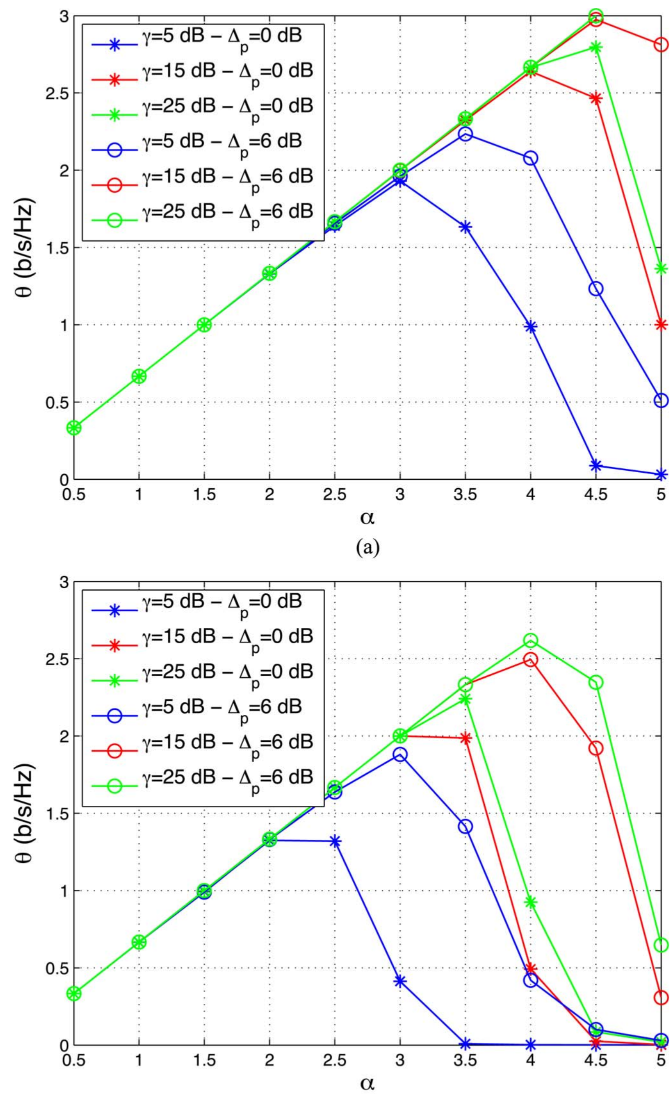


Fig. 7. Normalized throughput (θ) as function of the system load (α) for different SNR (γ): effect of power randomization with $\Delta_p = 0$ dB and $\Delta_p = 6$ dB on systems with $I = 10$ iterations at the receiver. (a) Rayleigh fading. (b) Log-normal fading.

TABLE II
MINIMUM GAP BETWEEN C AND θ ($I = 10$ AND $\Delta_p = 6$ dB)

Fading $\setminus$ α	1/2	1	3
Rayleigh	1.3 dB	0.7 dB	3.4 dB
Log-normal	1.7 dB	1.4 dB	4.0 dB

However, for overloaded systems with reasonable number of iterations, power randomizations is able to provide a significant benefit, especially in the medium SNR range. This is summarized in Tables I and II where the system with $\alpha = 3$ and $I = 10$ under log-normal fading reduces, through power randomization, the minimum capacity gap of 4.5 dB.

Fig. 7 confirms and enforces the discussion on power randomization. Focusing again on the linear throughput scaling as a function of the offered system load for the iterative receiver with $I = 10$ iterations operating at $\gamma = 15$ dB, it is apparent how power randomization is increasing the peak from 2.64 b/s/Hz to 2.98 b/s/Hz (resp. 2.00 b/s/Hz to 2.50 b/s/Hz) under Rayleigh fading (resp. log-normal fading).

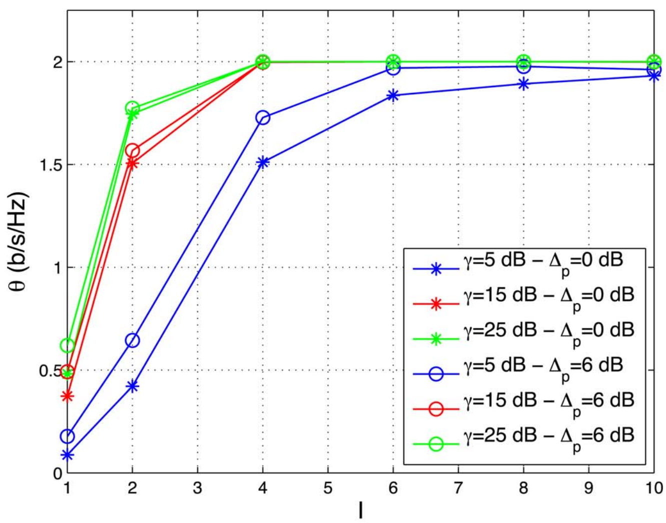
TABLE III
EFFECT OF POWER RANDOMIZATION ON THE PEAK NORMALIZED THROUGHPUT (θ IN B/S/HZ) AND CORRESPONDING SYSTEM LOAD (α) FOR ITERATIVE RECEIVERS WITH $I = 10$ ITERATIONS UNDER RAYLEIGH FADING

$\Delta_p \setminus \gamma$	5 dB	15 dB	25 dB
0 dB	$\theta = 1.93$; $\alpha = 3.0$	$\theta = 2.64$; $\alpha = 4.0$	$\theta = 2.80$; $\alpha = 4.5$
3 dB	$\theta = 1.96$; $\alpha = 3.0$	$\theta = 2.77$; $\alpha = 4.5$	$\theta = 2.89$; $\alpha = 4.5$
6 dB	$\theta = 2.23$; $\alpha = 4.0$	$\theta = 2.98$; $\alpha = 4.5$	$\theta = 3.15$; $\alpha = 5.0$

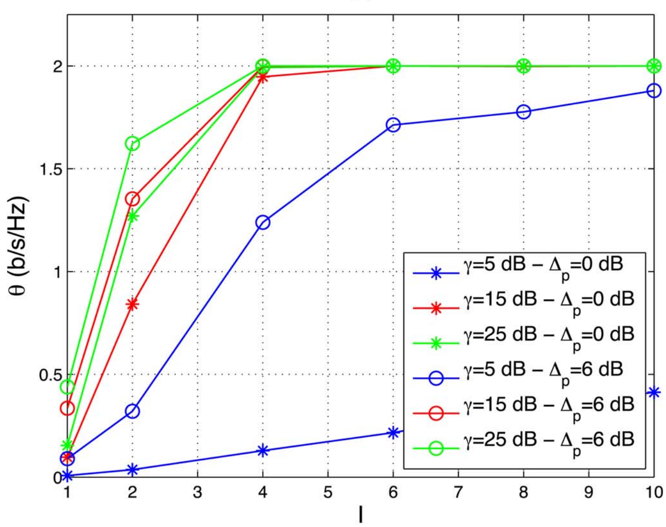
Additionally, in order to provide more insights on the behavior of the throughput as a function of the power-randomization range (Δ_p), Tables III and IV show the effect of three different ranges ($\Delta_p = 0$ dB, $\Delta_p = 3$ dB, and $\Delta_p = 6$ dB) on the achieved peak of the normalized throughput (θ) with corresponding system load (α). Again, it is apparent how the impact is more significant under log-normal fading than under Rayleigh fading.

TABLE IV
EFFECT OF POWER RANDOMIZATION ON THE PEAK NORMALIZED THROUGHPUT (θ IN B/S/HZ) AND CORRESPONDING SYSTEM LOAD (α) FOR ITERATIVE RECEIVERS WITH $I = 10$ ITERATIONS UNDER LOG-NORMAL FADING

$\Delta_p \setminus \gamma$	5 dB	15 dB	25 dB
0 dB	$\theta = 1.33$; $\alpha = 2.0$	$\theta = 2.00$; $\alpha = 3.0$	$\theta = 2.24$; $\alpha = 3.5$
3 dB	$\theta = 1.57$; $\alpha = 2.5$	$\theta = 2.29$; $\alpha = 3.5$	$\theta = 2.33$; $\alpha = 3.5$
6 dB	$\theta = 1.88$; $\alpha = 3.0$	$\theta = 2.50$; $\alpha = 4.0$	$\theta = 2.62$; $\alpha = 4.0$

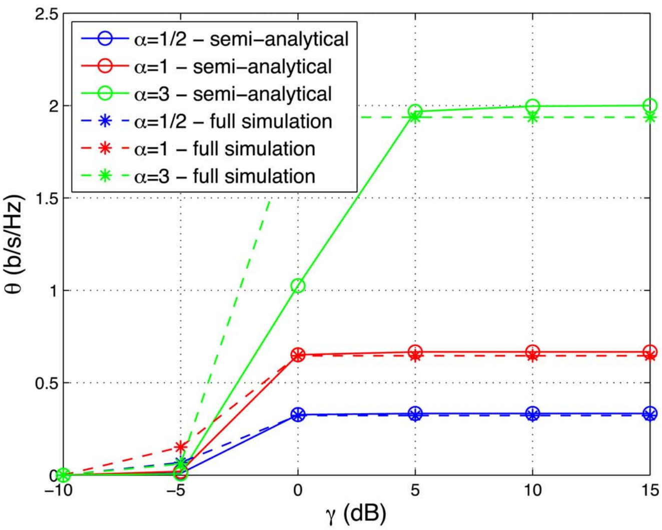


(a)

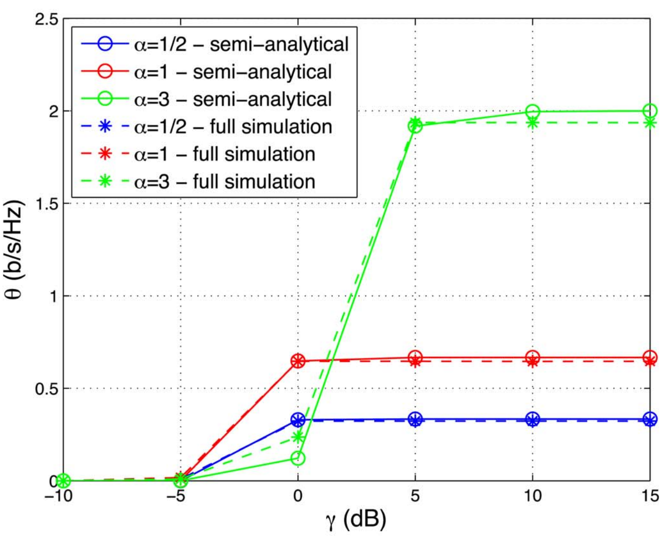


(b)

Fig. 8. Normalized throughput (θ) as function of the number of iterations (I) for different SNR (γ) and system load $\alpha = 3$: effect of power randomization with $\Delta_p = 0$ dB and $\Delta_p = 6$ dB. (a) Rayleigh fading. (b) Log-normal fading.



(a)

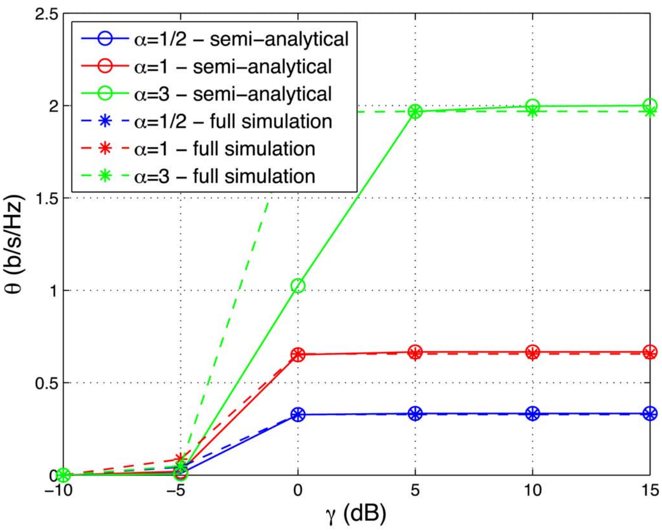


(b)

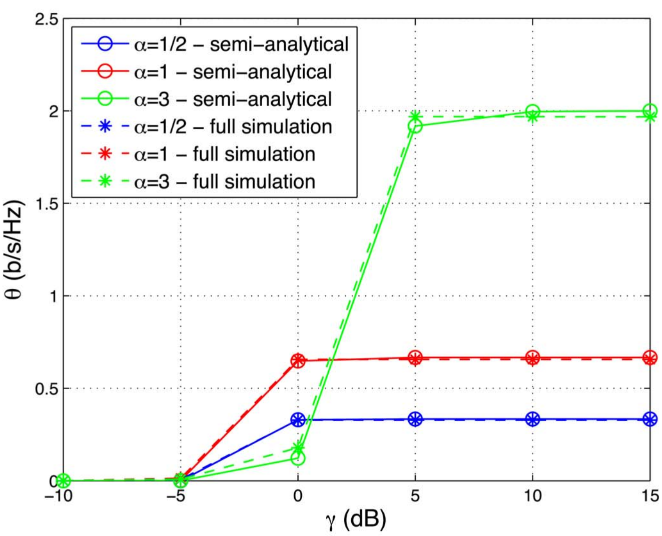
Fig. 9. Normalized throughput (θ) as function of the SNR (γ) for different system loads (α): comparison between the semi-analytical procedure and full-simulation results in the case of power randomization with $\Delta_p = 6$ dB for systems with $I = 10$ iterations at the receiver and $L = 288$ symbols/packet. (a) Rayleigh fading. (b) Log-normal fading.

Furthermore, it is interesting to consider employing power randomization as an alternative to increasing the number of iterations at the receiver in the case of overloaded systems, thus reducing the computational complexity and the decoding delay. Fig. 8 shows the normalized throughput (θ) with respect to the number of iterations (I) for a system load $\alpha = 3$ at $\gamma = 5$ dB (low SNR), $\gamma = 15$ dB (medium SNR) and $\gamma = 25$ dB (high SNR) in both scenarios with Rayleigh fading and log-normal fading. It is apparent how power randomization may be traded off with a few iterations at high SNR, thus not being crucial, while with much more iterations at low SNR, then exhibiting a large impact, which again is more significant under log-normal fading than Rayleigh fading.

Finally, in order to provide support to the reliability of the conclusions, the numerical results from the semi-analytical approach have been compared with those obtained with a full simulation (assuming a Poisson distribution for the packet arrivals) in the case of iterative receivers with power randomization for both scenarios with Rayleigh fading and log-normal fading. More specifically, Figs. 9 and 10 show the comparison in the case of $L = 288$ symbols per packet and $L = 576$ symbols per packet, respectively, in order to confirm the validity of the results for a large-enough range of packet lengths. Results from the semi-analytical procedure are generally slightly worse than those from a full simulation, thus we can claim that the semi-analytical approach, which is practically based on a Gaussian assumption for the multiuser interference, slightly over-estimates the detrimental effects of the residual interference.



(a)



(b)

Fig. 10. Normalized throughput (θ) as function of the SNR (γ) for different system loads (α): comparison between the semi-analytical procedure and full-simulation results in the case of power randomization with $\Delta_p = 6$ dB for systems with $I = 10$ iterations at the receiver and $L = 576$ symbols/packet. (a) Rayleigh fading. (b) Log-normal fading.

VI. Conclusion

Throughput performance of multiuser communications systems over fading channels have been analyzed by using a semi-analytical approach. Asynchronous transmissions is exploited for random access management and iterative receivers are considered for practical implementation issues. Results have been presented for Rayleigh and log-normal fading which may be considered realistic models for terrestrial and satellite wireless links, respectively. Power randomization has shown a significant impact on performance improvements, especially in the case of overloaded systems under log-normal fading scenarios.

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